First Exercise: Mechanical Energy:

A self supporting puck (S), assimilated to a particle of mass m = 1Kg, is launched from a point O with a speed $\vec{V_o} = 4\vec{i}$ with V_o= 4m/s, along the trajectory shown on figure-1. Given g = 10 ms^{-2} and OA = L = 1m. Frictional forces acting on (S,) are assimilated to a force $\vec{f}$ of constant magnitude that opposes the displacement along OAB. The position of (S) is determined by its abscissa along x'x with respect to the origin O.

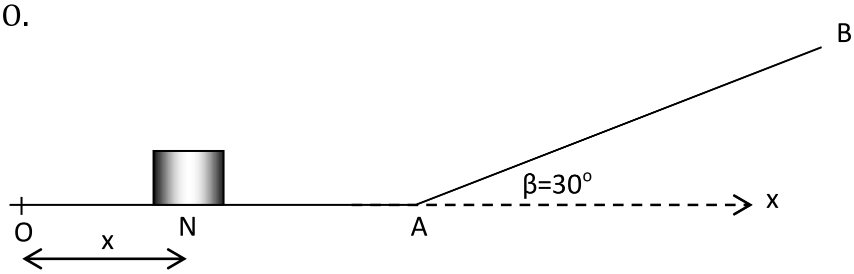
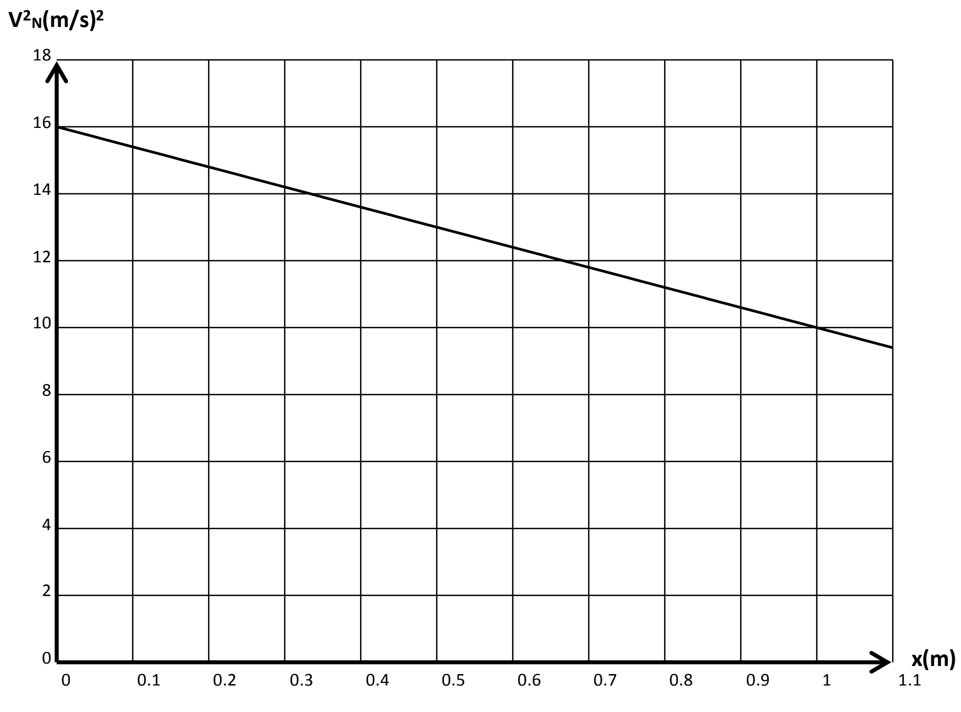


Figure-1

1) We want to study of the variation of the mechanical energy of the system (S, trajectory, Earth) between the launching point O and a point N with ON = x. show that $V_N^2 = \alpha.x + V_o^2$, where V_N is the speed of (S) at N and α is a constant to be determined in terms of f and m.
2) Deduce the expression of the acceleration of the puck.
3) The graph of figure-2 represents the variation of V_N^2 as a function of x. Calculate the slope of the straight line and deduce the value of f.
4) Determine, through two different methods, the speed of (S) at A.
5) Represent, without a scale, the forces acting on the puck and determine the $\Sigma\vec{F}_{ext}$.
6) Deduce the time needed by the puck to travel OA.
7) The system is isolated. Determine the variation of the microscopic energy along OA.
8) What distance does (S) travels along AB?



Second Exercise: Horizontal Elastic Oscillator [7 ½ pts]

A horizontal elastic oscillator is formed of an elastic spring of constant k = 40N/m having one end fixed to a support while the other end carries a particle of mass m = 100g. At a certain instant taken as an origin of time [$t_o = 0$], we record the variation of the abscissa $x = \overline{OG}$ where O is the equilibrium position of the center of mass G of the particle. The graph of the adjacent figure shows the variation of x as a function of time.
Take the level of G as a gravitational potential energy reference for the system [oscillator, Earth]. Take $g = 10 m/s^2$.

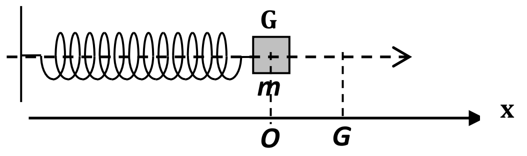


Figure-1

1) Write down the expression of the mechanical energy of the system [oscillator, Earth] at any time t in terms of m, x, k and the algebraic value V of the velocity of G.
2) The mechanical energy of the system is conserved. Why? Calculate, referring to the adjacent figure, its value.
3) Referring to the adjacent graph:
   a) Calculate at t = 0 the elastic potential energy of the oscillator.
   b) Deduce, at t = 0 , the kinetic energy of the particle.
   c) Deduce then the speed of the particle and the direction of its motion.
4) Derive the second order differential equation that describes the motion of **G**
5) Determine the expression of the period and calculate its value
6) Determine the time equation of motion.
7) Determine the speed of the particle and its direction of motion when G passes through the origin for the first time.
8) Determine, for x = 8cm, the speed of the particle.
9) Determine the expression of the linear momentum $\vec{P}$ of the particle at any instant and deduce the resultant of the forces acting on the particle during motion.
10) Verify the result of the above question by applying directly on the spring Hooke's law.

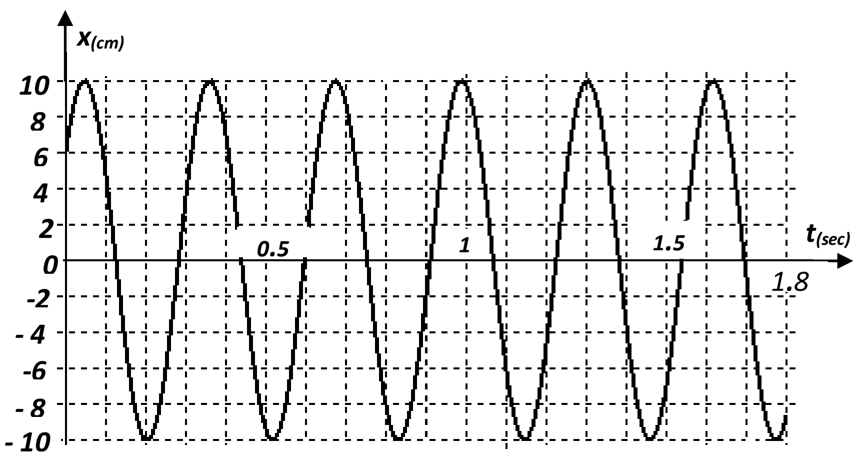
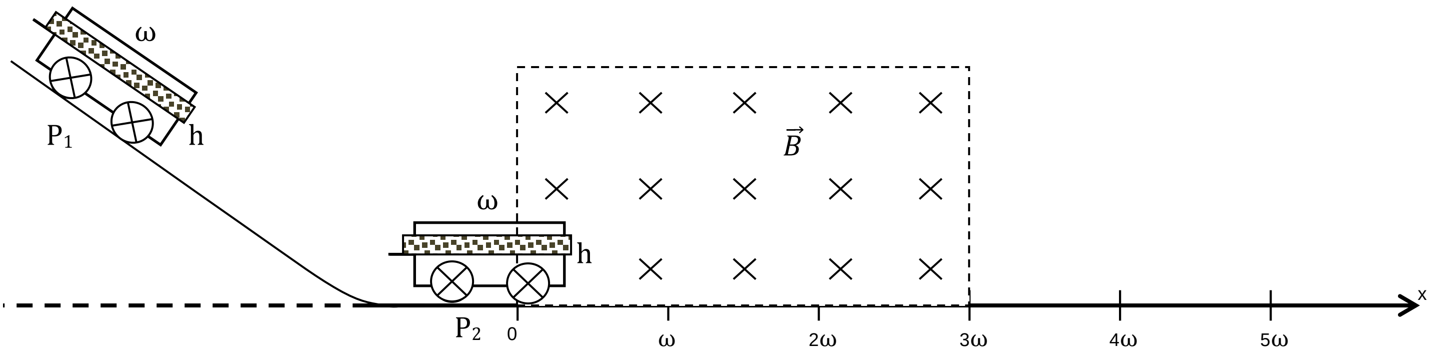


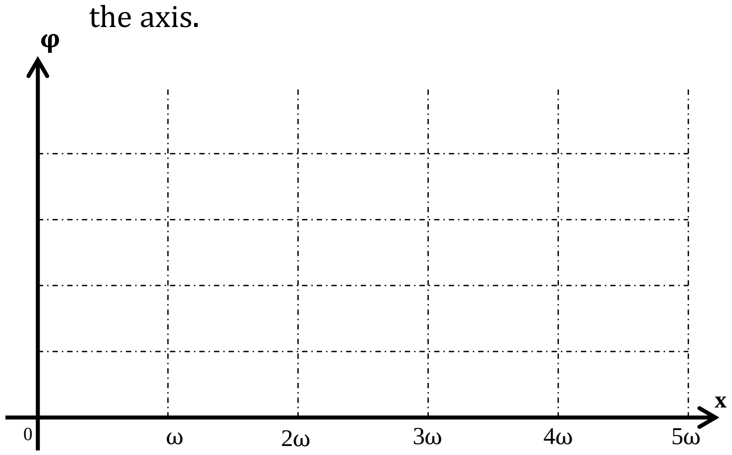
Figure-2

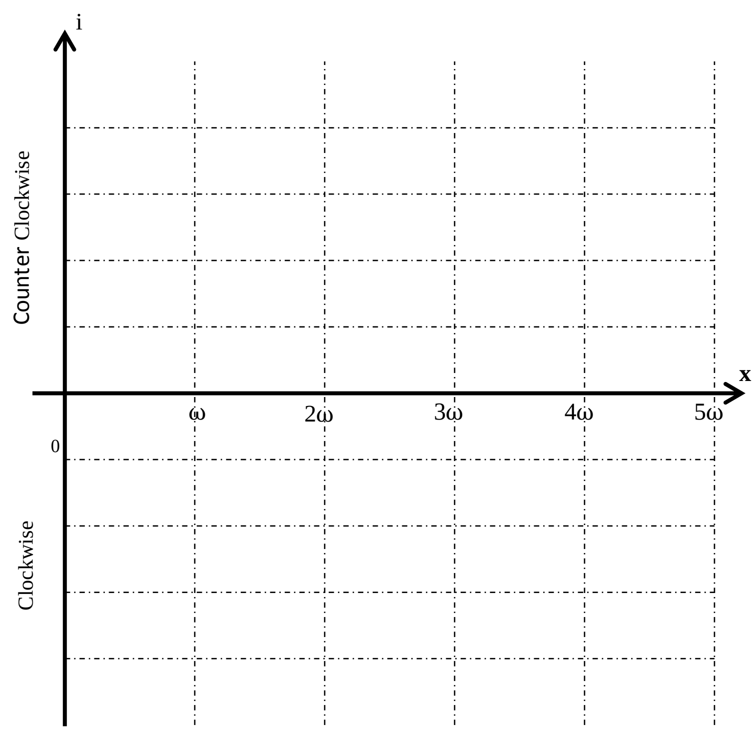
Third Exercise: Electromagnetic Induction:



A rectangular conducting loop of width ω, height h, and resistance R is mounted vertically on a nonconducting cart as shown above. The cart is released without initial speed from a position P_1 of the inclined part of the track making an angle $\propto$ with the horizontal part considered as a reference level for the gravitational potential energy. The length of the trip along the inclined part is d, and frictional forces are neglected along the whole track. As it attains the position P_2, the cart enters a uniform magnetic field of intensity B. The conducting loop is in the plane of the page and the magnetic field is directed into the page. The loop passes completely through the field with a constant speed. Express your answers in terms of the given quantities.

1) Determine the speed of the cart as it reaches the horizontal part of the track.
2) Determine the expression of the magnetic flux induced through the loop at an instant t where the loop enters the magnetic field.
3) Determine the expression of the induced e.m.f at any t.
4) Determine the expression of the induced current at any t.
5) Indicate the direction of the induced current as the cart enters the magnetic field.
6) Using the axis below, sketch the variation of the magnetic flux through the loop as a function of the horizontal distance x travelled by the cart, letting x = 0 be the position at which the front of the loop just enters the field. Label appropriate values on the axis.
7) Using the axis below, sketch the variation of the induced current through the loop as a function of the horizontal distance x travelled by the cart, letting x = 0 be the position at which the front of the loop just enters the field. Let counterclockwise current be positive and label appropriate values on the axis.



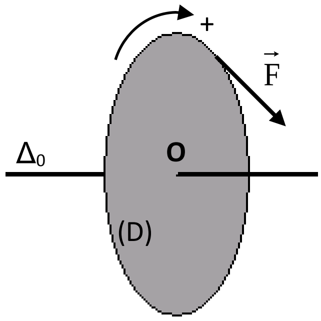


Fourth exercise: Mechanical Oscillations. (7,5pts)

The aim of this exercise is to determine the moment of inertia (I_0) of a disk with respect to its axis of revolution (Δ_0) through two different methods.

Given a disk (D) of center of mass (O). The disk (D), of radius R = 6cm and of mass m = 1kg, capable to rotate, in a vertical plane, about a horizontal axis (Δ_0) passing through its center (O).

Neglect all frictional forces and take $\pi = 1/0{,}32$ $g = 10 m/s^2$ and $\pi^2 = 10$.

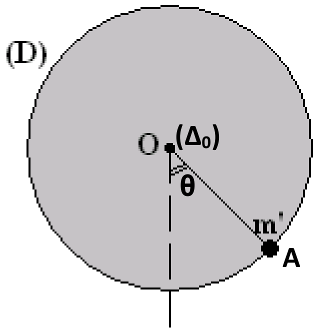


A) At an instant considered as origin of time $t_0 = 0$, a force $\vec{F}$, of constant magnitude F = 1,5N, is applied tangentially to the disk (D) initially at rest. At t, θ' is the angular speed of (D) about (Δ_0).

1) Give, at t, the expression of the angular momentum (σ) of the disk (D) in terms of I_0 and θ'.
2) Apply the theorem of angular momentum to determine the expression, in terms of time, of the angular velocity θ'.
3) Deduce the nature of the motion of disk (D).
4) At t = 2s, where the speed of rotation of the disk is 16 turns/s, the force $\vec{F}$ is stopped.
   a) Determine the nature of the motion of the disk (D) for t >2s.
   b) Determine the value of I_0.

B) The disk (D) is at rest. A particle of mass m' = m = 1kg is fixed at a point (A) of the periphery of (D). The compound pendulum, (P), thus formed, of center of mass G, is free to oscillate about (Δ_0).

The horizontal plane passing through (O) is considered as a reference level for the gravitational potential energy.



Initially at the position of equilibrium, the pendulum (P) is shifted by a small angle θ_m = 0,1rd then released without initial speed, at $t_0 = 0$. The position of the pendulum (P) is given at any t, by its angular abscissa θ that OG makes with the vertical passing by (O). Let θ' be the angular speed of (P) at any t.

For small values of θ consider: $\sin\theta = \theta_{rd}$ and $\cos\theta = 1-\frac{\theta^2}{2}$.

1) Show that: OG = a = R/2.
2) What is, in terms of R, the expression of the moment of inertia (I), of (P), with respect to (Δ_0)?
3) Show that the moment of inertia, of the pendulum (P), with respect to (Δ_0) is: $I = I_0 + 36\times10^{-4}$. (I and I_0 in kgm^2)
4) Establish the expression of the mechanical energy of the system (P, Earth) in terms of I, m, m', θ, θ' and R.
5) The mechanical energy of the system (P, Earth) is conserved. Why? Calculate its value.
6) Establish the differential equation which governs the motion of (P).
7) Deduce, in terms of I_0, the expression of the period (T) of the pendulum (P).
8) The time taken to achieve 20 complete oscillations is Δt = 12s. Determine I_0.

C) The moment of inertia of a homogeneous disk, of radius R and of mass m, with respect to an axis perpendicular to its plane and passing through its center is $I_0 = \frac{1}{2}mR^2$. Is the disk (D) homogeneous? Justify.

First exercise(7 ½ pts) horizontal elastic oscillator		
1.	M.E = K.E + P.E_g + P.E_e = ½ mV^2 + 0 + ½ kx^2	0.5
2.	M.E is conserved since the amplitude of the motion is constant. At any time : M.E = ½ kX_m^2 , with X_m = 10 cm from the graph , we get: M.E = = ½ (40)$(0.1)^2$ = 0.2J	0.75
3.a	At t = 0 , x_o = 6cm, P.Ee = ½ (40)$(0.06)^2$ = 0.072J	0.5
b	K.E_o = M.E – P.E = 0.2 – 0.072= 0.128J	0.5
c	K.E_o = ½ mV^2 then V^2= 2(0.128)/0.1 = 2.56 thus V_o = 1.6m/s at t = 0 the graph shows an increasing function then the velocity[derivative] is positive	0.75
4.a	M.E is constant then its derivative w.r. t is zero thus mVV' + kxx' = 0 but V = x' ≠ 0 and V' = x'' then : x'' +(k/m)x = 0	0.5
b	The solution of this equation is sinusoidal of the form x= $X_m$sin (ωt+ϕ) provided ω^2= k/m $T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{k}}$ = 6.28 /20 = 0.314s	0.75
c	x= $X_m$sin (ωt+ϕ) , Xm = 0.1m , ω = 20 rad/s and for t = 0 , x = 0.06 and V is positive so 0.06 = 0.1 sinϕ thus sinϕ = 0.6 and then ϕ = 0.6435 rad or π- 0.6435 V = ωX_m cos(ωt+ϕ) thus Vo = ωX_m cosϕ but Vo is positive then ϕ is acute = 0.6435 rad x = 0.1 sin (20t + 0.6435) , V = 2 cos(20t + 0.6435)	0.75
5	At that instant, x = 0 and decreasing then V is maximum= ωX_m =2m/s and negative	0.75
6	For x = 8cm = 0.08m, sin (ωt+ϕ) = 0.8, cos (ωt+ϕ) = = 0.6 thus V = 2(0.6) = 1.2m/s	0.75
7	$\vec{P} = m\vec{V} = 0.2\cos(20t + 0.6435)\ \vec{i}$, $\vec{F} = \frac{\vec{dp}}{dt} = -4\sin(20t + 0.6435)\ \vec{i}$	0.5
8	*Applying Hooke's law resultant force is the tension thus* $\vec{T} = -k\vec{x} = -(40)0.1\sin(20t + 0.6435) = -4\sin(20t + 0.6435)$	0.5

Ex-4	**Oscillations mécaniques.**	
A-1	$\sigma = I_0\theta'$	0,25
A-2	$\frac{d\sigma}{dt} = \sum Moments = M_{\vec{mg}} + M_{\vec{R}} + M_{\vec{F}} = I_0\frac{d\theta'}{dt} = M_{\vec{F}} = F.R \Rightarrow \frac{d\theta'}{dt} = \frac{FR}{I_0} = cte \Rightarrow \theta' = \frac{FR}{I_0}t.$	1
A-3	θ' = f(t) est une fonction de premier degré de temps alors le mouvement du disque est uniformément accéléré.	0,25
A-4	**a.** Pour t >2s; $\sum M = 0$; le mouvement de rotation ultérieur du disque est alors uniforme.	0,25
	b. $\theta' = \frac{FR}{I_0}t \Rightarrow I_0 = \frac{FR}{\theta'}t = \frac{1{,}5\times 0{,}06\times 2}{16\times 2\times \pi} = 1{,}8\times 10^{-3}$ kgm².	0,75
B-1	$\overrightarrow{OG} = \frac{m\overrightarrow{OO} + m'\overrightarrow{OA}}{m+m'} \Rightarrow OG = a = \frac{mR}{2m} = \frac{R}{2}.$	0,5
B-2	$I = I_0 + I_{m'} = I_0 + m'R^2$.	0,5
B-3	m' = 1kg alors $I = I_0 + 1\times(0{,}06)^2 = I_0 + 36\times10^{-4}$kgm².	0,5
B-4	$E_m = E_C + E_{PP} = \frac{1}{2}I\theta'^2 + (m+m')g(-a\cos\theta) = \frac{1}{2}I\theta'^2 - (m+m')g\left(\frac{R\cos\theta}{2}\right).$	0,75
B-5	Les forces extérieures ne travaillent pas alors $E_m = cte = E_{0m} = -(m+m')g\left(\frac{R\cos\theta}{2}\right) = -0{,}597J$	0,75
B-6	$\frac{dE_m}{dt} = 0 \Rightarrow I\theta'' + mgR(\sin\theta)\theta' = 0$ comme $\sin\theta = \theta_{rd}$ alors : $\theta'' + \frac{mgR}{I}\theta = 0.$	0,5
B-7	La solution de l'équation différentielle, établie dans (**B-6**), est de la forme : $\theta = \theta_m\sin(\omega_0 t+\phi)$ En remplaçant θ(t) et θ''(t) dans $\theta'' + \frac{mgR}{I}\theta = 0$ on obtient $\omega_0^2 = (mgR)/I$ alors $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mgR}} = 2\pi\sqrt{\frac{I_0 + R^2}{mgR}}$.	0,5
B-8	$T_0 = \frac{12}{20} = 0{,}6s \Rightarrow$ $\frac{T_0^2}{4\pi^2} = \frac{I_0 + R^2}{mgR} \Rightarrow I_0 = \frac{T_0^2 mgR}{4\pi^2} - R^2 = \frac{(0{,}6)^2\times 1\times 10\times(0{,}06)}{40} - (0{,}06)^2 = 1{,}8\times10^{-3}kgm^2$	0,5
C	$I_0 = \frac{1}{2}mR^2 = 0{,}5\times1\times(0{,}06)^2 = 1{,}8\times10^{-3}kgm^2$ qui est égale à la valeur trouvée du moment d'inertie (I_0) du disque (**B-9**). Alors le disque (D) est homogène.	0,5